MATH 304-03 Test II Spring 2013 Name ____________

Show all your work - use backs of pages if needed.

[16] 1. Let V be a vector space with $S \subseteq V$. Define the following:

(i) S is a spanning set for V.

(ii) S is a linearly independent subset of V.

(iii) S is a basis for V.

(iv) If V has a finite spanning set, dimension of V.

[18] 2. Let V be a vector space.

(i) State the principle of Independence Extension (hypothesis + conclusion).

(ii) State the principle of Span Preservation (hypothesis + conclusion).

(iii) Let $V = \mathbb{R}^3$, $I = \left\{ \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}, \begin{pmatrix} 4 \\ 3 \\ 17 \end{pmatrix} \right\}$ and $u = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$.

(a) Is $u \in \operatorname{Span}(I)$? ______ (show your work)

(b) Which conclusion is valid: X $\operatorname{Span}\{I \cup \{u\}\} = \operatorname{Span}(I)$
or
Y $I \cup \{u\}$ is linearly independent? ? ______

[10] 3. Let V be a vector space with a finite spanning set. Give your opinion about the following statements with True or False.

______ (i) All finite spanning sets of V have the same number of ~~ele~~ elements.

______ (ii) All linearly independent subsets of V have finitely many elements.

______ (iii) V has a finite basis.

______ (iv) All bases for V have the same number of elements.

______ (v) A spanning set for V with $\dim V$ number of elements is linearly dependent.

______ (vi) If W is a subspace of V with $\dim W = \dim V$, then $V = W$.

[40] 4. Let $V = P_2$, the vector space of polynomials of degree ≤ 2. As you may assume, $X = \{x^2, x^2+x, x^2+x+1\}$ and $Y = \{x^2+1, x+1, x^2-2x-2\}$ are bases for V. Consider them ordered bases on this page.

(i) Calculate $\ _X(I)_Y$ (here I is the identity transformation. $Iv = v$)

(ii) Calculate $\ _Y(I)_X$

(iii) Let $F: V \to V$ be a linear transformation for which it is known $\ _Y[F]_X = \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & 2 \\ 1 & 1 & 1 \end{pmatrix}$

(a) If $v \in V$ such that the coordinates of v w.r.t. X are $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = [v]_X$, calculate $F(v)$.

(b) Calculate $\ _X[F]_Y$

[38] 5. Let V be the subspace of space differentiable functions from $\mathbb{R}$ to $\mathbb{R}$ spanned by $B = \{x\cos x,\ x\sin x,\ \cos x,\ \sin x\}$

Assume, as is true, B is a linearly independent set.

(i) $\dim V =$ _____.

Let $W = \{f \in V \mid f(0) = 0\}$.

(ii) Show W is a subspace of V.

(iii) Find a set of 3 linearly independent vectors in W.

(iv) $\dim W =$ _____.

Consider $D: V \longrightarrow V$ by $D(f) = f'$ (the derivative)

(v) Calculate ${}_B(D)_B$

Let $D^2 = D \circ D$, the composition of functions.

(vi) Calculate ${}_B(D^2)_B$

(vii) Calculate ~~Calc~~ ${}_B[D^2]_B [x\cos x]_B$

[41] 6. Let $V = P_2$, $W = \mathbb{R}^3$ and the linear transformation $F : V \to W$ by $F(a_0 + a_1 x + a_2 x^2) = \begin{pmatrix} a_0 + a_1 + a_2 \\ 2a_0 - a_1 + 2a_2 \\ a_0 + a_2 \end{pmatrix}$

Assume, as it is true, $X = \{x+1,\ x^2+1,\ x^2+x+1\}$ and $Y = \left\{ \begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}1\\1\\1\end{pmatrix} \right\}$ are ordered bases for V and W respectively.

(i) Calculate ${}_Y(F)_X$

(ii) Find a basis for $F(V)$, the image of V under F.

(iii) Find a basis for $\ker F$.

(iv) Is F onto? ____

(v) Is F one-to-one? ____