This exam is worth 150 pts. Show all your work - you may use backs of page.

30] 1. Find all solutions (if any) to the following system of equations.

$$x - y + z = 4$$
$$2x + y + 3z = 1$$
$$7x - y + 9z = 14$$

Solution set: $x =$

$y =$

$z =$

Which variables are basic? ________

Which variables are free? ________

Which are the pivot positions? ________

Is $x = 2$, $y = 1$, $z = 3$ a solution? ________

Let $X$ be a set of matrices; $X=\{A,B,C,D,E\}$

where $A=\begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \end{pmatrix}$, $B=\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$, $C=\begin{pmatrix} 1 & 3 \\ 2 & 2 \\ 3 & 1 \end{pmatrix}$, $D=\begin{pmatrix} 1 & 0 & 1 & 0 \\ 2 & 1 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}$

and $E=\begin{pmatrix} 1 & 2 & 3 & 2 & 1 \\ 4 & 4 & 1 & 1 & 1 \end{pmatrix}$.

(i) Assume one matrix in $X$ is invertible. Which one is it? ____

(ii) Find two matrices $S$, $P$ in $X$ with $S \neq P$ for which $3S-2P^T$ is defined.

$S=$ ____, $P=$ ____, $3S-2P^T=$

(iii) Find 3 matrices $S$, $P$, $Q$ in $X$ such that $S \neq P$ but $S+PQ$ defined. $S=$ ____ $P=$ ____, $Q=$ ____

$S+PQ=$

(iv) Find matrices $S$, $P$ in $X$ such that $SP$ is a $3\times 5$ matrix

$S=$ ____

$SP=$

$P^TS^T=$

$P=$ ____

3. (i) Find the matrix $E$ such that if $A$ is a $4 \times n$ matrix then $EA$ is the matrix obtained by performing the elementary row operation of replacing row 2 by row 2 minus 3 times row 3.

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(ii) Let $A = \begin{pmatrix} 7 & 5 \\ 4 & 3 \end{pmatrix}$. Write $A$ and $A^{-1}$ as a product of elementary matrices.

$A =$

$A^{-1} =$

[illegible] 1. Suppose $V$ and $W$ are vector spaces and $T : V \to W$ is a funct

(i) domain of $T$ = ______ (ii) codomain of $T$ = ______

(iii) What is the definition of "$T$ is a linear transformation"

(iv) Now suppose $V = \mathbb{R}^3$, $W = \mathbb{R}^4$, $T$ is a linear transformation, and it is known $T\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}0\\1\\0\\1\end{pmatrix}$, $T\begin{pmatrix}1\\1\\0\end{pmatrix} = \begin{pmatrix}1\\1\\0\\0\end{pmatrix}$ and $T\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$

Write the standard matrix for $T$

(v) For $T$ as given in (iv), calculate $T\begin{pmatrix}2\\-1\\4\end{pmatrix}$.

5. (i) What is the definition of "$U$ is a subspace of vector space $V$"?

(ii) True or False : ____ $\{(u, v) \mid u, v$ are even integers$\}$ is a subspace of $\mathbb{R}^2$.

____ $\{(u, v, w) \mid u + v = -w\}$ is a subspace of $\mathbb{R}^3$

____ If $U$ is a subspace of $V$ and $L$ is a linear transformati[on]

6. [illegible] to reduced echelon form is correc[t]

$$\left(\begin{array}{ccccc|c} 1 & -2 & -3 & 0 & 1 & a \\ 0 & 1 & 2 & 1 & 0 & b \\ 2 & 0 & 3 & 1 & 1 & c \\ 5 & 0 & 7 & 4 & 3 & d \\ 0 & 0 & 0 & 0 & 1 & e \end{array}\right) \quad \text{reduces to} \quad \left(\begin{array}{ccccc|c} 1 & 0 & 0 & 5 & 0 & 3a+6b-c-2e \\ 0 & 1 & 0 & 7 & 0 & 4a+9b-2c-2e \\ 0 & 0 & 1 & -3 & 0 & -2a-4b+c+e \\ 0 & 0 & 0 & 0 & 1 & e \\ 0 & 0 & 0 & 0 & 0 & -a-2b-2c+d \end{array}\right)$$

$C$ is the coefficient matrix.

(i) Find a vector $x$ (if it exists), $x \in \mathbb{R}^5$ with $Cx = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 2 \\ 0 \end{pmatrix}$

(ii) Find a vector $y$ (if it exists) $y \in \mathbb{R}^5$ with $Cy = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 5 \\ 2 \end{pmatrix}$

(iii) Find (if it exists) a nonzero vector in the null space of $C$.

(iv) Express (if possible) $\begin{pmatrix} 0 \\ 1 \\ 1 \\ 4 \\ 0 \end{pmatrix}$ as a linear combination of $\left\{ \begin{pmatrix} 1 \\ 0 \\ 2 \\ 5 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ 2 \\ 3 \\ 7 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \\ 3 \\ 1 \end{pmatrix} \right\}$

7. For $A = \begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{pmatrix}$, calulate (if it exists) $A^{-1}$