Math 220 - Calculus f. Business and Management - Worksheet 34

Solutions for Worksheet 34 - Mixed Integration Methods

1: $\int (3x^2 - 8x)(x^3 - 4x^2 + 2)^{12}\, dx$

Solution for #1:

We set $u = x^3 - 4x^2 + 2$ *so that* $\frac{du}{dx} = 3x^2 - 8x$. *Therefore,* $du = 3x^2 - 8x\, dx$. *Now we plug everything in:*

$$\int (3x^2 - 8x)(x^3 - 4x^2 + 2)^{12}\, dx \;=\; \int u^{12}\, du \;=\; \frac{u^{13}}{13} + C \;=\; \frac{(x^3 - 4x^2 + 2)^{13}}{13} + C$$

2: $\int \left(\frac{3}{\sqrt[4]{x}} + e^x \right) dx$

Solution for #2:

We write the first term so that we have no radicals: $3/\sqrt[4]{x} = 3x^{-1/4}$. *Then we integrate.*

$$\int (3x^{-1/4} + e^x)\, dx \;= 3 \cdot \frac{4}{3} x^{3/4} + e^x + C \;=\; 4x^{3/4} + e^x + C$$

3: $\int (x + 1) e^{3x^2 + 6x}\, dx$

Solution for #3:

Use a u-substitution. Set $u = 3x^2 + 6x$, *then* $\frac{du}{dx} = 6x + 6 = 6(x + 1)$. *Therefore,* $\frac{1}{6} du = (x + 1)\, dx$. *Now plug everything in and evaluate.*

$$\int (x + 1) e^{3x^2 + 6x}\, dx \;= \frac{1}{6} \int e^u\, du \;=\; \frac{e^u}{6} + C \;=\; \frac{1}{6} e^{3x^2 + 6x} + C$$

4: $\int x^3 \ln x\, dx$

Solution for #4:

Use integration by parts. Set $u = \ln(x)$ *so that* $du = \frac{1}{x} dx$. *Set* $dv = x^3 dx$ *so that* $v = \int x^3 dx = \frac{x^4}{4}$. *Now,*

$$uv - \int v \cdot du \;=\; \ln(x) \cdot \frac{x^4}{4} - \int \left(\frac{1}{x} \right) \left(\frac{x^4}{4} \right) dx \;=\; \frac{x^4 \ln(x)}{4} - \frac{1}{4} \int x^3\, dx \;=\; \frac{x^4 \ln(x)}{4} - \frac{1}{16} x^4 + C$$

5: $\int \sqrt{7t-5}\,dt$

Solution for #5:

Use u*-substitution. Let* $u = 7t-5$*. Then* $du = 7 \cdot dt$*. Therefore,* $dt = \frac{1}{7}du$*. We now substitute in and integrate:*

$$\int \sqrt{7t-5}\,dt \;=\frac{1}{7}\int u^{1/2}\,du \;=\; \left(\frac{1}{7}\right)\left(\frac{2}{3}\right)u^{3/2} \;+\; C \;=\; \frac{2}{21}(7t-5)^{3/2} \;+\; C$$

6: $\int \frac{1+6q^3}{q}\,dx$

Solution for #6:

This one seems tricky because it is a rational function, but you can simply carry out the division by q *to make things much easier for yourself. Always try to make the integral easier for yourself if possible!*

$$\int \frac{1+6q^3}{q}\,dq \;=\; \int \frac{1}{q}+6q^2\,dq \;=\; \int q^{-1} \;+\; 6q^2\,dq \;=\; \ln|q| \;+\; 6\cdot\frac{1}{3}q^3 \;+\; C \;=\; \ln|q|+2q^3 \;+\; C$$

7: $\int \frac{\sqrt[3]{x}}{\sqrt[5]{x}}\,dx$

Solution: for #7

Again this one seems tricky, but observe that we can combine the top and bottom of the fraction. Remember that when you divide, you subtract the exponents. We do this below:

$$\int \frac{\sqrt[3]{x}}{\sqrt[5]{x}}\,dx \;=\; \int x^{1/3}\cdot x^{-1/5}\,dx \;=\; \int x^{(5/15-3/15)}\,dx \;=\; \int x^{2/15}\,dx \;=\; \frac{15}{17}x^{17/15} \;+\; C$$

8: $\int \frac{2p^3+3}{(p^4+6p)^5}\,dx$

Solution for #8:

We will set $u = p^4+6p$ *so that* $\frac{du}{dp} = 4p^3+6 = 2(2p^3+3)$*. Thus,* $\frac{1}{2}du = (2p^3+3)\,dp$

This will work nicely to substitute in now:

$$\int \frac{2p^3+3}{(p^4+6p)^5}\,dp \;=\frac{1}{2}\int u^{-5}\,du \;=\; \frac{1}{2}\cdot\left(\frac{-1}{4}\right)\cdot u^{-4} \;+\; C \;=\; \frac{-1}{8}(p^4+6p)^{-4}+C = \frac{-1}{8(p^4+6p)^4} \;+\; C$$

9: $\int (4x-6)e^{5x}\,dx$

Solution for #9:

We use integration by parts here. We will set $u = 4x-6$ *so that* $du = 4\,dx$ *and we will also set* $dv = e^{5x}$ *so that*

$v = \frac{1}{5}e^{5x}$. Notice to integrate dv you need to use a substitution for $5x$ (this is omitted here). Now we can integrate:

$$uv - \int v\,du \;= (4x-6)\cdot\frac{e^{5x}}{5} - \int \frac{e^{5x}}{5}\cdot 4\cdot dx \;= \qquad \frac{e^{5x}}{5}(4x-6) - \frac{4}{5}\int e^{5x}\,dx$$

$$= \frac{e^{5x}}{5}(4x-6) - \frac{4}{5}\frac{e^{5x}}{5} \;+\; C \;=\; \frac{e^{5x}}{5}(4x-6-\frac{4}{5}) + C = (4x - \frac{34}{5})\frac{e^{5x}}{5} \;+\; C$$

10: $\int \frac{4x}{2x^2-1}\,dx$

Solution for #10:

We will do a substitution and set $u = 2x^2-1$. Then $\frac{du}{dx} = 4x$ so that $du = 4x\,dx$. We are ready to integrate now.

$$\int \frac{4x}{2x^2-1}\,dx \;=\; \int \frac{du}{u} = \ln|u| + C = \ln|2x^2-1| \;+\; C$$

11: $\int \frac{x^2+\sqrt{x}}{3x}\,dx$

Solution for #11: *We use our brains here and notice that we can carry out the division by $3x$ to change the expression to a form we can easily deal with. This is done below:*

$$\int \frac{x^2+\sqrt{x}}{3x}\,dx \;=\; \int \frac{x}{3} + \frac{x^{1/2}}{3x}\,dx \;=\; \int \frac{x}{3} + \frac{x^{-1/2}}{3}\,dx \;=\; \frac{x^2}{2\cdot 3} + \frac{2\cdot x^{1/2}}{3} \;+\; C \;=\; \frac{x^2}{6} + \frac{2\sqrt{x}}{3} \;+\; C$$

12: $\int \frac{\ln(3x+2)}{3x+2}\,dx$

Solution for #12: *This one is tricky. We will take $u = \ln(3x+2)$, because we know that $\frac{du}{dx} = \frac{1}{3x+2}\cdot 3$. This means $\frac{du}{3} = \frac{dx}{3x+2}$. This is what we see in the function. Notice that choosing $u = 3x+2$ would be very hard to deal with here (try doing it and see how far you get). Now:*

$$\int \frac{\ln(3x+2)}{3x+2}\,dx \;=\; \frac{1}{3}\int u\,du = \frac{1}{3}\frac{u^2}{2} \;+\; C \;=\; \frac{[\ln(3x+2)]^2}{6} \;+\; C$$