# Math 220 - Calculus f. Business and Management - Worksheet 8

## *Solutions for Worksheet 8 - Composite and Multivariable Functions*

### *Composition of functions*

***Exercise 1:*** *Compose these pairs of functions into a single function*

$$\textbf{\textit{1A}}: C(q) = 5q + 2500; \quad q = q(p) = 1500 - p; \quad C(q(p)) = ?$$
$$\textbf{\textit{1B}}: f(y) = \ln(y+5); \quad y = y(x) = 6x^2 - 3x + 5; \quad f(y(x)) = ?$$
$$\textbf{\textit{1C}}: V(r) = (4/3)\pi \cdot r^3; \quad r = r(t) = 3t; \quad V(r(t)) = ?$$
$$\textbf{\textit{1D}}: R(q) = 100q - \ln(q/100); \quad q = q(p) = \sqrt{250-p}; \quad R(q(p)) = ?$$

***Solution to #1:***

***1A***: *We replace $q$ with $q(p) = 1500 - p$ in the expression $C(q)$:*
$C(q(p)) = 5(1500-p) + 2500 = 10000 - 5p$

***1B***: *We replace $y$ with $y(x) = 6x^2 - 3x + 5$ in the expression $f(y)$:*
$f(y(x)) = \ln(6x^2 - 3x + 5 + 5) = \ln(6x^2 - 3x + 10)$

***1C***: *We replace $r$ with $r(t) = 3t$ in the expression $V(r)$:*
$V(r(t) = (4/3)\pi \cdot (3t)^3 = (4 \cdot 27/3)\pi \cdot t^3 = 36\pi \cdot t^3$

***1D***: *We replace $q$ with $q(p) = \sqrt{250-p}$ in the expression $R(q)$:*
$R(q(p)) = 100\sqrt{250-p} - \ln(\sqrt{250-p}/100)$

### *Decomposition of functions*

***Exercise 2:*** *Decompose these functions into two (or more) separate functions.*

$$\textbf{\textit{2A}}: \ f(g(x)) = \sqrt{\ln x}; \quad \textbf{\textit{2B}}: \ v(h(t)) = e^{6t}; \quad \textbf{\textit{2C}}: \ s(m(h)) = (h^3 + 4h - 7)^5;$$
$$\textbf{\textit{2D}}: \ a(b(x)) = e^{5x+7}; \quad \textbf{\textit{2E}}: \ h(s(w(x))) = \ln\sqrt{4x^2 - 3x};$$

*Solutions to A:*

***2A***: $f(g(x)) = \sqrt{\ln x} \ \rightsquigarrow \ u = g(x) = \ln x, \ f(u) = \sqrt{u};$
***2B***: $v(h(t)) = e^{6t} \ \rightsquigarrow \ u = h(t) = 6t, \ v(u) = e^u;$
***2C***: $s(m(h)) = (h^3 + 4h - 7)^5 \ \rightsquigarrow \ a = m(h) = h^3 + 4h - 7, \ s(a) = a^5;$
***2D***: $a(b(x)) = e^{5x+7} \ \rightsquigarrow \ z = b(x) = 5x + 7, \ a(z) = e^z;$
***2E***: $h(s(w(x))) = \ln\sqrt{4x^2 - 3x} \ \rightsquigarrow \ u = w(x) = 4x^2 - 3x, \ v = s(u) = \sqrt{u}; \ h(v) = \ln(v);$

### *Multivariable Word Problems*

***Exercise 3:*** *The length of a rectangle is 3 times as long as its width. Express the area of the rectangle three ways. First as a function of the length and width. Second as a function of the width only. Third as a function of the length.*

***Solution to #3:***

*First we need some variable names: Let $w$ stand for the width and let $l$ stand for the length of the rectangle. We have the following functional relationships between $w$ and $l$:*

$$l = L(w) = 3w, \qquad w = W(l) = l/3.$$

*The area of a rectangle is always width $\times$ length. Here are the three area functions $A_1(l,w), A_2(w), A_3(l)$ we are looking for:*

$$\begin{aligned}
\textbf{\textit{3a}} &: A_1(l,w) \,=\, l \cdot w,\\
\textbf{\textit{3b}} &: A_2(w) \,=\, A_1(L(w),w) \,=\, A_1(3w,w) \,=\, 3w \cdot w \,=\, 3w^2,\\
\textbf{\textit{3c}} &: A_3(l) \,=\, A_2(l,L(w)) \,=\, A_2(l,l/3) \,=\, l \cdot l/3 \,=\, l^2/3.
\end{aligned}$$

***Exercise 4:*** *The height of a cylinder is 5 times its radius. Express the volume of the cylinder as a function of the height and radius. Then express it as a function of the height only. Finally express it as a function of the radius only.*

***Solution to #4:***

*First we need some variable names: Let $r$ stand for the radius, $a$ for the area of the circular base, $h$ for the height of the cylinder and let $v$ stand for the volume of the cylinder. We have the following functional relationships between $r$, $h$ and $a$:*

$$a \,=\, \pi \cdot r^2, \qquad h \,=\, H(r) \,=\, 5r, \qquad r \,=\, R(h) \,=\, h/5.$$

*We remember that the volume of a cylinder is area $a$ of the circular base $\times$ height $h$, i.e., $v \,=\, \pi \cdot r^2 h$. Here are the three volume functions $V_1(h,r), V_2(h), V_3(r)$ we are looking for:*

$$\begin{aligned}
\textbf{\textit{4a}} &: V_1(h,r) \,=\, \pi \cdot r^2 h,\\
\textbf{\textit{4b}} &: V_2(h) \,=\, V_1(h,R(h)) \,=\, V_1(h,h/5) \,=\, \pi \cdot (h/5)^2 h \,=\, \pi \cdot h^3/25,\\
\textbf{\textit{4c}} &: V_3(r) \,=\, V_1(H(r),r) \,=\, V_1(5r,r) \,=\, \pi \cdot r^2 5r \,=\, 5\pi \cdot r^3.
\end{aligned}$$

*By the way: did you notice that you also could have computed $V_3(r) \,=\, V_2(H(r)) \,=\, V_2(5r)$?*

***Exercise 5:*** *Using the measurements from exercise 4, express the total area of the cylinder (sides and top and bottom) three ways.*

***Solution to #5:***

*Again we start by assigning variable names. We shall reuse those of the previous exercise: Let $r$ stand for the radius, $a$ for the area of the circular base, $l$ for its perimeter length, $h$ for the height of the cylinder and let $s$ stand for the surface area of the cylinder. The functional relationships between $r$, $h$ and $a$ remain the same as above. In addition we also can express $l$ as a function of the radius $r$:*

$$a \,=\, \pi \cdot r^2, \qquad l \,=\, \pi \cdot 2r, \qquad h \,=\, H(r) \,=\, 5r, \qquad r \,=\, R(h) \,=\, h/5.$$

*We remember that the surface area $s$ of a cylinder is twice the area $a$ of the circular base (top and bottom of the cylinder surface) plus circle perimeter length $l \times$ height $h$, i.e., $s \,=\, 2\pi \cdot r^2 \,+\, 2\pi r \cdot h$. Here are the three surface area functions $S_1(h,r), S_2(h), S_3(r)$ we are looking for:*

$$\begin{aligned}
\textbf{\textit{5a}} &: S_1(h,r) \,=\, 2\pi \cdot r^2 \,+\, 2\pi r \cdot h \,=\, 2\pi r(r+h),\\
\textbf{\textit{5b}} &: S_2(h) \,=\, S_1(h,R(h)) \,=\, S_1(h,h/5) \,=\, 2\pi(h/5)(h/5+h) \,=\, 2\pi(h/5)\frac{h+5h}{5} \,=\, (12/25)\pi \cdot h^2,\\
\textbf{\textit{5c}} &: S_3(r) \,=\, S_1(H(r),r) \,=\, S_1(5r,r) \,=\, 2\pi r(r+5r) \,=\, 2\pi \cdot r \cdot 6r \,=\, 12\pi \cdot r^2.
\end{aligned}$$

*Of course you also could have computed $S_3(r) \,=\, S_2(H(r)) \,=\, S_2(5r)$ instead amd gotten the same result.*